**NBSIR 81-2410** 

# Measurement of Ion Current Density At Ground Level in the Vicinity of High Voltage DC Transmission Lines

U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards
National Engineering Laboratory
Center for Electronics and Electrical Engineering
Electrosystems Division

December 1981

100 .U56 81-2410 1981 c. 2

Department of Energy Office of Electric Energy Systems 12th Street & Pennsylvania Ave., N.W. Mail Stop 3344 Washington, DC 20461



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MEASUREMENT OF ION CURRENT DENSITY AT GROUND LEVEL IN THE VICINITY OF HIGH VOLTAGE DC TRANSMISSION LINES

R. H. Mcknight, F. R. Kotter, and M. Misakian

Sensors for measuring vertical current density at ground level near high voltage dc (HVDC) transmission lines are subject to error when the sensor is not in the ground plane. The magnitude of this error, for guarded and unguarded sensors, has been investigated using both dc electric fields with space charge and ac electric fields in a parallel plate facility. For conditions like those expected under HVDC transmission lines, the results obtained using ac and dc methods agreed to within experimental uncertainty. The measured errors are as large as 25 percent for guarded sensors and significantly larger for unguarded sensors. Data for various sensor elevations and guarding are presented in graphs to aid the designer.

Key words: high voltage dc; measurement errors; transmission lines; vertical current density; Wilson plates.

## 1. INTRODUCTION

It is not economically feasible to design high voltage dc (HVDC) transmission lines with conductors of sufficiently large diameter that minor surface defects and adhering foreign particles will not result in significant corona generation at normal operating voltages. In the usual operating mode, the transmission line consists of two conductors (or conductor bundles) of opposite polarity and balanced with respect to ground. In the immediate vicinity of the negative conductor, the positive ions in the corona discharge are attracted to the conductor and neutralized. The same is true for the negative ions near the positive conductor. Ions of the same polarity as the conductor are repelled and tend to migrate along electrostatic field lines to the other conductor or to ground depending on their initial locations.

The ions produced are so-called small ions with mobilities of the order of  $10^{-4}$  m²/V·s. Some recombination occurs during migration and some ions grow in size by attachment to other molecules and aerosols. Wind forces acting on the ions can predominate over the electrical forces and carry larger ions over considerable distances—well out of significant influence of the electric field. Thus, both conduction and convection currents exist in the presence of wind. In theory at least, these components of the total current density can be independently, although indirectly, measured. Measurements of the wind speed (u) and the net space-charge density ( $\rho$ ) permit calculation of the convection current density,  $j_u$  ( $j_u = u_p$ ). Similarly, measurements of the electric field (E) and of the

conductivity ( $\lambda$ ) permit calculation of the conduction current density, j<sub>e</sub> (j<sub>e</sub> =  $\lambda$ E).

No techniques have been developed for direct measurement of either the conduction or convection components of the current density at an arbitrary point in space; however, in principle at least, measurement of the vertical component of the conduction current density at the ground surface is quite simple [1]. 1

Measurements of the current density at ground level are of value in estimating the electric potential to which insulated persons or objects which intercept the ion current can be elevated. Under conditions where the current density can be considered conduction current density ( $j_e$ ) only, a measurement of the electric field (E) and an assumption regarding the value of the weighted average ion mobility (K) permits an estimate of the ion density  $\rho_p$  ( $\rho_p = j_e/KE$ ) of a given polarity.

Current to a conducting plate lying in the ground plane, but insulated from ground, can be measured with electrometer circuits which require negligible rise of the probe plate potential above the ground reference. Current-density measuring devices of this type have been used for many years in atmospheric electricity research under the name "Wilson plates" [3-6]. The current density levels of interest in that area of research are orders of magnitude lower than those frequently encountered in the vicinity of HVDC transmission lines. The larger current simply eases the requirement on the current measuring instrument when ion currents from power lines are to be measured.

The accuracy customarily sought in Wilson plate measurements of atmospheric electricity has not been high, since there is more interest in changes due to weather, diurnal variations, etc., than in absolute magnitudes [7]. The principal sources of error (excluding instrumental uncertainty) identified and discussed in the atmospheric research literature are: (1) the change in the plate potential resulting from the flow of charge to it, (2) the displacement current introduced by moving clouds of space charge, and (3) the lack of coplanarity between the Wilson plate and the local ground plane [8]. The first two items have been studied, but the coplanarity requirement apparently has not been investigated by atmospheric scientists. One earlier study, concerned with currents induced in objects located under ac transmission lines, does deal with the question of unguarded plates under ac conditions [9].

It is important to ascertain how large these various sources of error are so that reliable estimates can be made of the uncertainty in measurements made near HVDC transmission lines. If the sources of

Numbers in brackets refer to the references listed at the end of this report.

error are well understood, then corrections can be made to data which will improve the overall accuracy of the measurement.

This report focuses on the third type of error mentioned above. Specifically, miniature, Wilson-plate-type, current sensors are used under controlled laboratory conditions to explore the errors which result from use of guarded and unguarded plates at various elevations above the ground. Data were obtained for both ac and dc fields of varying intensity.

## 2. EXPERIMENTAL PROCEDURE

## 2.1 General

The vertical conduction current density  $(j_e)$  at the surface of a Wilson plate is given by

$$j_e = \rho_p KE$$
 (1)

where the various symbols have been previously defined. The actual current measured with the plate is the integral of  $j_e$  over the surface of the plate. If the sensing plate is raised above the ground plane, then the electric field distribution at the surface of the plate changes. In particular, as is well known from the theory of electrostatics, the field increases due to geometrical enhancement, and if K and  $\rho_p$  remain constant, the measured  $j_e$  will increase. This results in an error in the measure of  $j_e$  which is due to the spatial location of the sensor.

In an analogous fashion, a Wilson plate sensor exposed to an ac field will have induced in it a displacement current which is also proportional to the integral of the electric field over the surface of the plate. For the ac conditions, an increase in the displacement current would be observed as the sensor is raised above the ground plane. This increase is due to the geometrical enhancement of the field by the sensor just as for the situation with dc fields and ions.

One would expect that the effect of raising the current sensor out of the ground plane would be the same for the measurement of displacement current in a space-charge free region at 60 Hz as for measurement of the ion current with direct voltage if the following assumptions are valid: (1) in the dc case the ions follow the field lines, and (2) the presence of the space charge in this case does not distort the field pattern.

In this experiment, measurements were made both for ac fields and for dc fields with space charge. The measurements of displacement current at 60 Hz can be made with greater precision and accuracy than comparable measurements with dc conduction currents. The ac measurements can also be made with a much simpler experimental arrangement than that required for the dc field plus ions. If, as is shown to be the

case later, there is agreement between measurements taken using the two methods then the ac technique can be used to determine the behavior of a given sensor. .

The facility used for producing ion currents in the presence of a dc field is shown schematically in figure 1 and has been previously described in detail [10]. The apparatus permits the establishment of controllable levels of ion current density  $(j_{\rm e})$  between the parallel plates in the presence of a dc electric field which can be varied independent of the current density. The electric field and spacecharge density in the region between the plates can be calculated in terms of experimentally determined quantities. The facility can also be operated with 60-Hz excitation with no space charge.

The current-sensing plates used in these studies are illustrated in figure 2. They were constructed from 0.157-cm thick copper-clad fiber glass sheets. On six of them a central section of the copper was isolated from an outer band (henceforth referred to as a guard ring) by a narrow slot (about 0.5 mm wide) milled through the copper as indicated. The nominal dimensions are tabulated in the figure.

## 2.2 AC Measurements

Using 4 kV, 60-Hz excitation of the parallel-plate structure, with a plate separation of approximately 0.5 meter, data were taken with each of the plates listed in figure 2. These data consisted of the displacement current to the plates as a function of height above the ground plane. Figure 3 is representative of the data obtained with one set (A-D) of plates [11]. Plotted here is the percentage increase in displacement current above that measured with the sensor in the ground plate. The value of a guard ring in reducing this increase is evidenced by the substantially greater slope observed for the unguarded sensor. The ac data were very reproducible and the data shown are typical of those obtained in a single run.

The maximum uncertainty in these results is a few percentage points. However, relative values are considered correct to within a few tenths of one percent. The important sources of uncertainty are the effects of the leads to the plates and perturbations of objects nearby the parallel plate system. Manually inserted spacers, all of which were smaller in cross section than the sensor plate, were used to raise the sensor plates above the ground plate. It was determined that the spacer size did not affect the results.

#### 2.3 DC Measurements

The parallel plate system was also operated to produce a do field plus ions. In these measurements the electric field and current density at the ground plate were varied over a wide range and data

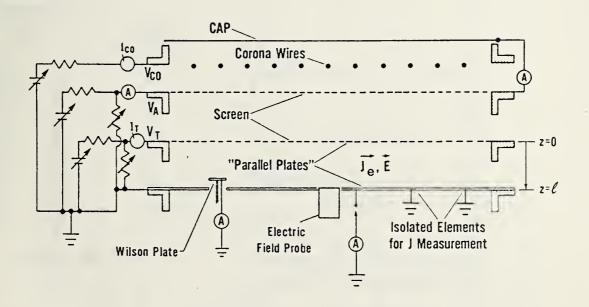


Figure 1. Schematic view of apparatus. The configuration shown is appropriate for generating an electric field with positive space charge. The top and bottom plates of the parallel plate structure are located in the planes z=0 and z=1. The corona wires are oriented parallel to one another and are perpendicular to the plane of the figure. Operation of the apparatus is described in detail in Ref. 10.

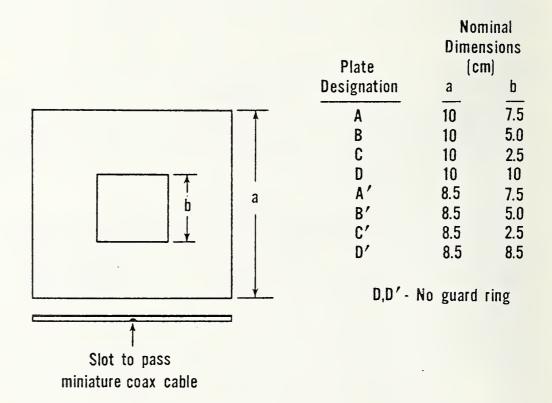


Figure 2. Wilson plates used in the present study.

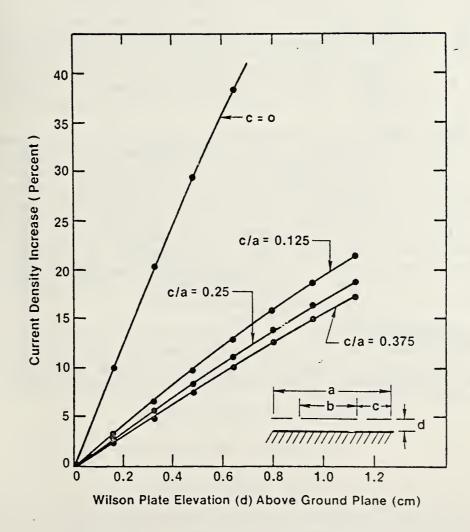


Figure 3. Percentage increase in 60-Hz displacement current density for plates A-D (fig. 2) as function of Wilson plate elevation above ground plane. Smooth lines connect related data points.

were acquired for some of the sensors shown in figure 2. Current densities ranged from 50 nA to 3  $\mu A$  and field values were varied from 4 to 35 kV/m.

This system is much less stable and reproducible when operated in a dc mode than with ac excitation. As a result, special measures had to be taken to obtain useful results.

For the parallel plate system operating to produce a dc conduction current, it was not possible to reproduce operating conditions during a data run unless the various electrode voltages were left unchanged. For the purpose of this study, a current-sensing plate was slowly raised, by means of a motor drive, out of the plane of the ground electrode in figure 1. Its elevation above the plane was determined using a cathetometer. In this way, a stable operating condition could be maintained as the elevation of the plate was varied.

The current to the sensing plate was measured with a commercial electrometer which drove a strip chart recorder. One observer marked the chart at times when predetermined elevations were reached as indicated by a second observer using the cathetometer. A second current probe mounted on the ground plate of the assembly was used to monitor the ion current density being introduced into the parallel-plate structure and small adjustments of the voltages applied to the ion-generating section were made, when necessary, to maintain a constant current to this probe during a series of observations. Under these conditions, the electric field and current density at the ground plate were constant during a data run.

Even under the controlled conditions and with the precautions indicated above, the variability associated with the ion production and transport made it impossible to obtain the same precision achieved in the ac measurements. Some indication of the spread in data acquired is evident in figure 6.

#### 3. RESULTS AND DISCUSSION

## 3.1 AC Measurements

The percent increase of current density shown in figure 3 can be thought of as a measurement error as the Wilson plate is elevated above the ground plane. Figure 4 summarizes such measurement errors in normalized form for all the Wilson plate probes using 60-Hz electric fields. The curves presented here are smooth curves drawn through points taken from data similar to that shown in figure 3. The data are shown in terms of parameters related to the geometry and location of the sensor plate. The importance of even a small guard ring is clearly evident. The width of the guard ring becomes much less important as the sensor is brought near the ground plane (larger a/d). For small a/d values, which correspond to larger elevations

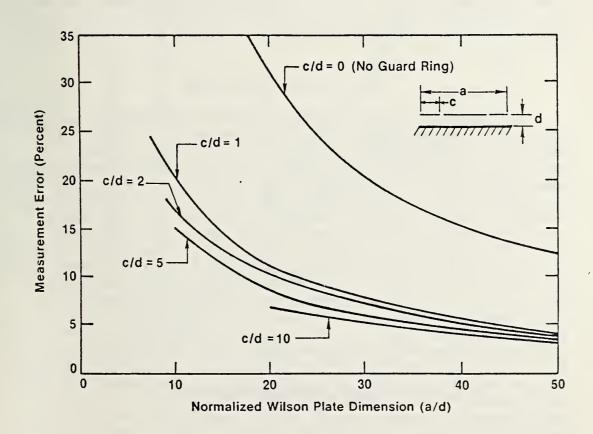


Figure 4. Wilson plate measurement errors obtained with 60-Hz electric fields as a function of guard ring width and elevation above the ground plane.

for a given sensor, a wider guard ring can reduce the error by several percent. It is worth noting that for a one-meter square Wilson plate 2 cm above the ground plane (a/d = 50), the error for a guard ring as wide as 0.20 m is still of the order of 4 percent.

Deno [9] experimentally determined the enhancement factors for an unguarded plate above the ground surface under a 60-Hz high voltage test line. Direct comparison of his results with those reported here are possible only for a/d = 16, where he observed an enhancement of about 36 percent—to be compared with the 38 percent extrapolated from the figure 4 data. This is considered good agreement, especially in view of the fact that his experiment was designed to explore the problem of metal roofs with a resultant emphasis on lower values of a/d.

## 3.2 DC Fields Plus Ions

The precision of the dc current density measurements, as indicated above, was considerably less than that obtained with an ac field. However, it was sufficient to conclude that, except for the case of a probe without a guard ring, for low values of space-charge density the ac and dc results were the same within the accuracy of the measurements. Figure 5 shows some typical data of the percentage increase in current density versus probe height above the ground plane for two different dc operating conditions. An earlier ac result is included for comparison. The data in figure 5 are presented for two different values of the parameter  $\Delta E/E_g$  which is proportional to space-charge density in the neighborhood of the ground plane. Here  $\Delta E$  is  $E_g$  -  $E_l$  where  $E_g$  and  $E_l$  are the calculated values [10] of the unperturbed electric field strength at the ground plane and one centimeter above the ground plane, respectively. The variable  $\Delta E/E_{\alpha}$  then serves as a convenient measure of the contribution of the špace charge near the current sensor to the electric field (and thus to the current density) at the surface of the sensor--relative to the total electric field existing at that surface. In general, the agreement between ac and dc results diminishes, however, as the space-charge density is increased. This is clearly seen for the two cases shown in the figure.

In figure 6 the percent increase in ion current density for a selected current sensor for a wide range of values of  $\Delta E/E_g$  is indicated. The scatter in the data alluded to earlier is evident here. Each point represents a current density increase calculated from data such as that shown in figure 3 for different electrical operating conditions. Each of these operating conditions is characterized by a known set of applied potentials, plate spacing, average current density to the ground plate, and a measured electric field at the ground plate (fig. 1). This information is used to determine  $\Delta E/E_g$ . In general, for a given field, as the current density increases, the parameter  $\Delta E/E_g$  increases.

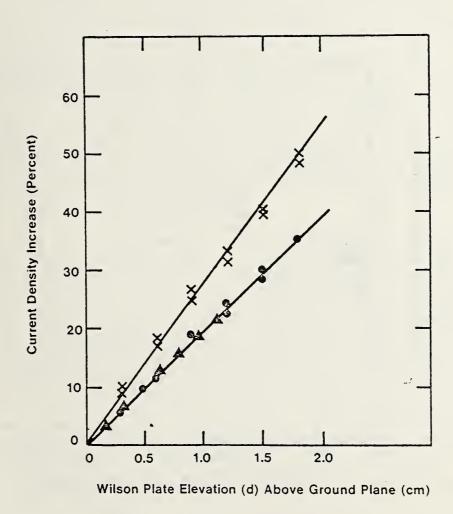


Figure 5. Percentage increase in current density as function of Wilson plate (probe A) elevation and space-charge contribution to electric field.  $\bullet$  -  $\Delta E/E_g$  = 0.004; x -  $\Delta E/E_g$  = 0.024;  $\Delta$  - ac values.

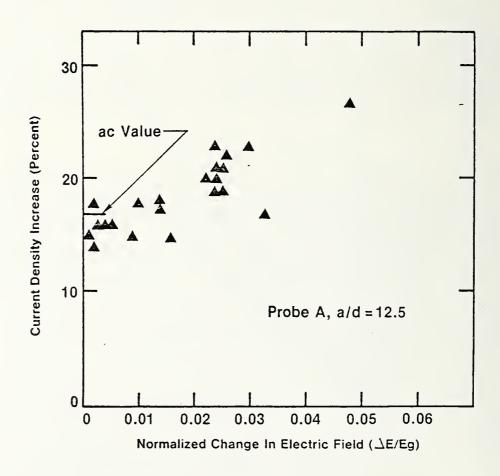


Figure 6. Percentage increase of current density as function of space-charge density near the ground plane for probe A and a/d=12.5 (see fig. 2). While there is much scatter in the data, an upward trend is evident.

## 3.3 Analysis of DC Results

An accurate theoretical description of the ion current intercepted by the model Wilson plates has not been developed because of cost and time factors. It is worthwhile, however, to discuss qualitatively the results observed for different values of  $\Delta E/E_g$ . If the contribution to the total electric field from the space charge is very small, the electric field is nearly uniform, and the dc current density data should agree with the 60-Hz results as the Wilson plate is raised above the ground plane. That is, the ions will travel along electric field lines and be collected by the sensing element; the increase in the ion current as the height is increased will be functionally the same as that for the number of field lines terminating on the sensing element. Because the charge density is initially small, the increase of the electric field (as the sensing element is raised) at and near the sensing element due to the charge density can be considered small and neglected.

When the space charge makes a large contribution to the electric field, the change in electric field at the Wilson plate surface as the probe is elevated differs from the ac case. Qualitatively the data in figure 5 suggest that the field strength increases at a faster rate, as a function of probe elevation, than in the space-charge free case because the now significant charge density is increased due to the field perturbation. This means that a more rapid increase in measurement error should be observed for larger charge densities and the figure 6 data confirm this expectation. Additional comparisons between the dc and ac results are shown in figure 7. The solid curves are identical with those of figure 4 and show the results of the ac measurements series. The individual points plotted show the results of the dc series of measurements using  $\Delta E/E_q$  as a parameter. Except for the case of the unguarded sensor, the do data points for  $\Delta E/E_q < 0.004$  lie very close to the corresponding ac curves; however, that is not true for data points corresponding to larger values of that parameter.

Reliable measurements of the polar charge density in the vicinity of HVDC transmission lines are not yet available; however, the results of two independent calculations are in hand [12,13]. These agree reasonably well with one another and predict values for the parameter  $\Delta E/E_q$  (scaled to a one-meter square Wilson plate with  $\Delta E$  being the field difference over a 10-cm distance) in the range 0.002 to 0.006. On this basis it is concluded that the ac curves of figure 4 for guarded Wilson plates may be applicable to measurements in the vicinity of HVDC transmission lines.

It should be noted that with the unguarded sensor, the dc data points in figure 7 for all values of  $\Delta E/E_g$  lie well below the ac curve. The explanation for this is readily evident from a consideration of the field pattern shown in figure 8. Field lines which terminate on the underside of the sensing plate contribute

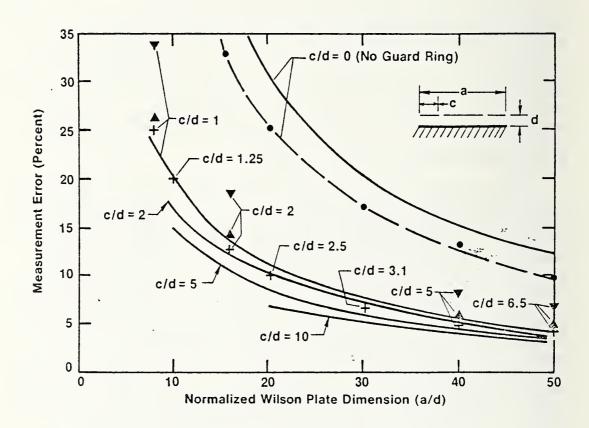


Figure 7. Errors in Wilson plate current density measurements as a function of guard ring width and elevation above the ground plane. The solid curves are results of measurements with alternating current. The  $\Delta E/E_g$  parameter (see main text) values for the dc data plotted are as follows: + - 0.004;  $\Delta$  - 0.014;  $\nabla$  - 0.024;  $\Phi$  - 0.004.

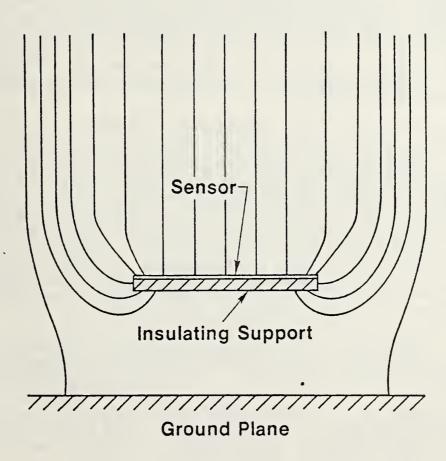


Figure 8. Schematic view of flux lines terminating on a sensor plate located above the ground plane.

to the induced current (the ac case) but ions which traverse these field lines (the dc case) are prevented from reaching the sensing plate by the insulating support. If the resistivity of the insulation is sufficiently high, presumably a charge will accumulate on the surface of the insulation and distort the field. If the resistivity is not so high as to permit significant charge accumulation, the ion current associated with this fringing field will divide between the sensing plate and the ground plane. In either case the results for the unguarded plate with ac cannot be expected to apply when the plate is used for measurement of ion current. This same effect would of course apply to the quarded sensors as well, but since most field lines which terminate on the underside are near the edge, their effect will be negated by the quard ring. If the unquarded sensor had been constructed entirely of a conducting material, the ac and ion current measurements would be expected to agree.

## 3.4 Location of Ground Plane

Even when care is taken to assure that the plane of the current sensor very nearly coincides with the ground surface, a question arises with respect to the location of the "effective" ground plane. At first thought this might be expected to be a function of the nature of the soil and of its water content. However, as the analysis given below indicates, only in exceptional situations, e.g., if the sensor were to lie on solid rock, would there be justification for differentiating between the surface of the ground and the effective ground plane in measurements of the normal component of current density at the ground level.

Consider the "true" ground plane to lie a distance, h, below and parallel to the ground surface and assume that the soil between the two planes is homogeneous. Then the ground surface becomes an equipotential surface which differs from "true" ground potential by a voltage, Eh. A grounded Wilson-plate-type current sensor then distorts the field at the ground surface--the extent of the distortion being directly related to the potential difference between the Wilson plate and the plane in which it lies.

The electric field strength in the soil is E =  $j\rho_S$  where j is the current density and  $\rho_S$  the soil resistivity. Experimentally determined representative values of soil resistivity given in an IEEE standard [14] range from a minimum of 3.4 to a maximum of 4580 ohmmeters. If one assumes a uniform current density of  $10^{-6}$  A/m<sup>2</sup> incident on a ground surface where the resistivity has the approximate value, 4600 ohmmeters, the field in the soil is 0.00460 V/m. If the true ground plane is a meter below the ground surface, the grounded Wilson plate differs in potential from the surface on which it rests by 4.6 millivolts.

This seems to be a worst case and indicates that the resulting field distortion is negligible, and the error introduced is likewise negligible. Note that in the case of a Wilson plate off the ground surface only 2 cm in a field of 10 kV/m, the plate potential differs from the space potential by 200 volts. For a guarded one meter Wilson plate the error so introduced (from fig. 4) is about 4 percent.

## 4. MODELS FOR A WILSON PLATE NOT IN THE GROUND PLANE

As noted in the previous section, accurate model calculations which predict the experimental results observed during the present study cannot be readily performed. Two simplified models are developed in this section, however, which qualitatively predict some of the observed results.

## 4.1 Current Density Measurements: 60-Hz Electric Fields

An accurate theoretical treatment for the problem of induced currents in the model Wilson plates has not been attempted. However, an estimate of the expected error can be obtained from a consideration of the field perturbation produced by an oblate spheroidal boss on the surface of a plane on which a uniform electric field is incident. The enhancement of the field at the center of the surface of such a boss has been tabulated as a function of the ellipsoidal parameters [15]. Figure 9 provides a comparison of the experimental results obtained using the small Wilson plates with calculations for two rather arbitrarily chosen spheroids used as approximations to the actual geometry. The geometry of the boss relative to the Wilson plate is shown in the figure. These results are considered of value in that they provide order-of-magnitude confirmation that the experimental results obtained are in agreement with those which can be expected from theoretical considerations. A theoretical model which provides an explanation of the trends observed in the measurements is developed below.

A Wilson plate located in the ground plane is shown schematically in figure 10(a) and is modeled approximately as a grounded conducting sphere with an isolated sensing element shown in figure 10(b). The sphere is located in the zero potential plane of a uniform 60-Hz electric field produced with parallel plates that are infinitely far away. Raising the Wilson plate above the ground plane is simulated by elevating the sphere above the zero potential plane, z=0. A general expression for the electric field at the surface of a sphere in a uniform field is given by [16]

$$E_{S} = [(\phi - V)/a + 3E_{O}\cos\theta] \sin \omega t, \qquad (1)$$

where  $\Phi$  is the space potential at the center of the sphere prior to the introduction of the sphere, V is the potential of the sphere,

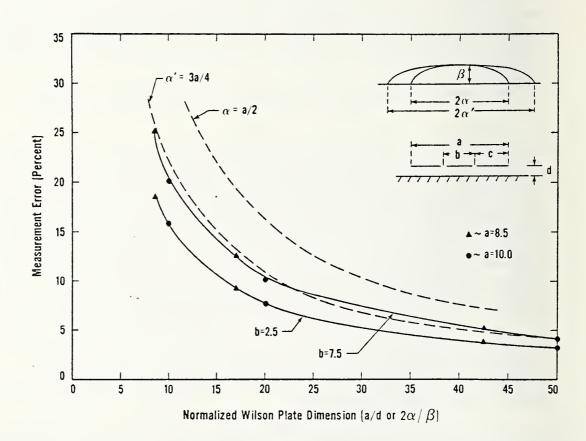
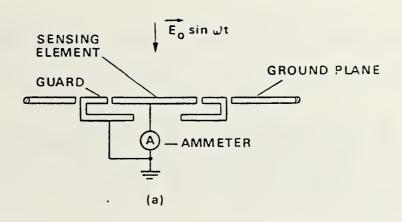


Figure 9. Comparison of experimental values with calculations for a prolate spheroidal boss. In the calculations presented,  $\beta$  = d and two values of  $\alpha$  were chosen:  $\alpha$  = a/2,  $\alpha'$  = 3a/4. For the experimental results shown, the data are indicated as  $\Delta$  - a = 8.5 cm;  $\bullet$  - a = 10 cm. Calculated curves shown as ...



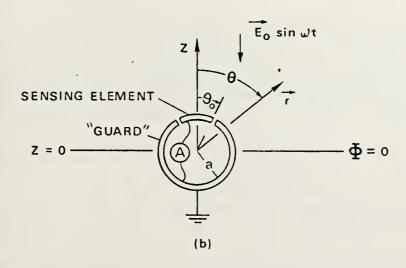


Figure 10. (a) Representation of a guarded Wilson plate located in the ground plane. (b) Schematic view of a spherical current sensor, modeled in the text.

a is the sphere radius,  $E_0$  is the peak value of the electric field,  $\theta$  is the polar angle in spherical coordinates, and  $\sin \omega t$  indicates the sinusoidal time dependence of the potentials and field. For a grounded sphere located in the zero potential plane, eq (1) reduces to

$$E = 3E_0 \cos \theta \sin \omega t. \tag{2}$$

As the sphere is raised above the zero potential plane, eq (1) becomes

$$E_{S} = [\Phi/a + 3E_{O} \cos\theta] \sin \omega t.$$
 (3)

When the sphere is raised, we assume that the equipotential surface remains flat, but theoretically the surface will be distorted near the sphere. Because the field is uniform,  $\Phi$  can be replaced with h  $E_0$  where h is the distance above the ground plane. Equation (3) can then be put into the form

$$E_S = 3 E_O (\sin \omega t) [\cos \theta + \gamma h],$$
 (4)

where  $\gamma$  = (1/3a). The factor in brackets in eq (4) describes how the field at a point on the surface of the sensing element is enhanced as the sphere, simulating the Wilson plate, is raised above the ground plane.

Noting that the surface charge density on the sensing element of the sphere is given by  $\sigma = \epsilon_0 E_S$ , where  $\epsilon_0$  is the permittivity of free space, it is possible to show [17] that the current induced in the sensing element defined by the angle  $\theta_0$  (fig. 10(b)) is given as a function of height by

$$I = 2\pi\omega a^2 \epsilon_0 E_0 (\cos \omega t) [h(1-\cos\theta_0)/a + (3/2) \sin^2\theta_0].$$
 (5)

Noting that the sensing area is given in spherical coordinates as

$$A = \int_{0}^{2\pi} \int_{0}^{\theta_0} a^2 \sin\theta \ d\theta d\phi = 2\pi a^2 (1 - \cos\theta_0), \qquad (6)$$

the current density, I/A, can be put into the form

$$J = J_0[1+\beta(\theta_0)h] \tag{7}$$

where  $J_0$  is the current density when h = 0,

$$J_0 = (3/2)\omega \varepsilon_0 E_0 \sin^2 \theta_0 (\cos \omega t)/(1-\cos \theta_0), \qquad (8)$$

and  $\beta(\theta_0)$  is given by

$$\beta(\theta_0) = 2(1-\cos\theta_0)/(3a \sin^2\theta_0). \tag{9}$$

The normalized rate of change of J as h is increased (i.e., as the "Wilson plate" is raised above ground plane) is just

$$\frac{1}{J_0} \frac{dJ}{dh} = \beta(\theta_0) . \tag{10}$$

Equations (9) and (10) predict that a graph of normalized current density (J/Jo) versus height above the ground plane (h) will (1) have a constant slope for a given guard ring width (fixed  $\theta_0$ ), and (2) show a greater slope (measurement error) if a Wilson plate with a smaller guard ring is used above the ground plane (e.g.,  $\beta(60^\circ)>\beta(30^\circ)$ ). Both predictions have been observed experimentally (fig. 3) suggesting that the approximations used in developing the model are not unreasonable. An expression analogous to eq. (10) for the dc case with space charge could not be developed.

## 5. CONCLUSIONS

As a result of these studies it is concluded that, in general, unguarded Wilson-plate-type current sensors should be used only if the sensor plane can be made to coincide very closely with the ground surface. More flexibility is available if the sensing plate is guarded. Figure 4 can then be used in the design of a current sensor as follows:

A decision as to the minimum elevation above the ground plane and the maximum lateral plate dimensions which are practical determines the value of a/d. The guard ring width required to reduce the error resulting from the non-zero value of d to a value considered acceptable can then be estimated by interpolation between the curves given.

It is also noteworthy that under most conditions there need be no concern over the question as to how far below the surface of the

ground the "true" ground surface lies. Finally, it was found that data which predict the Wilson plate measurement errors of 60-Hz displacement currents above the ground plane are also appropriate for predicting measurement errors for conduction current under HVDC transmission lines.

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NBS-114A (RE V. 2-AC)							
U.S. DEPT. OF COMM.	1. PUBLICATION OR	2. Performing Organ, Report No. 3.	Publication Date				
BIBLIOGRAPHIC DATA SHEET (See instructions)	NBSIR 81-2410		December 1981				
4. TITLE AND SUBTITLE							
Measurement of Ion	Current Density at Gr	ound Level in the Vicini	ty of High				
Voltage DC Transmi			•				
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5. AUTHOR(S)							
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E BEREORMING ORGANIZA	TION (If joint or other than NBS	see instructions)					
6. FERFORMING ORGANIZA	TION (II Joint of other than NBS	FA.	Contract Grant No77-01-6010				
NATIONAL BUREAU OF	STANDARDS	Tas	sk'No: ÄÖ18-EES				
DEPARTMENT OF COMM		8. T	ype of Report & Period Cover				
WASHINGTON, D.C. 2023	4						
9. SPONSORING ORGANIZAT	TION NAME AND COMPLETE A	DDRESS (Street, City, State, ZIP)					
Department of Ener							
Office of Electric							
	sylvania Ave., N.W.						
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10. SUPPLEMENTARY NOTE	:S						
· Document describes a	a computer program; SF-185, FIP	S Software Summary, is attached.					
11. ABSTRACT (A 200-word of	or less factual summary of most	significant information. If document i	includes a significant				
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12 KEY WORDS (Six to swall	ve entries: alphabetical order; ca	pitalize only proper names; and separ	rate key words by semicolons!				
high voltage dc; measurement errors; transmission lines; vertical current density; Wilson plates.							
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13. AVAILABILITY			14. NO. OF				
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